

Reliability Analysis of Stability of Sonapur Slope

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Abstract—Different methods of reliability are being employed for assessment of the probability of failure and the reliability of a slope. Probabilistic strategies include Monte Carlo Simulation method (MCS), First and Second Order Reliability Methods (FORM, SORM), Mean-Value First-Order Second-Moment method (MFOSM). GEO5, PLAXIS 2D are the software based on optimized slip surface and FEM respectively and are being used for obtaining FOS. Based on Rt software three methods of FOSM, FORM/SORM and MCS are used for obtaining the Reliability Index.

Keywords: reliability analysis, probability of failure, factor of safety, normal distribution.

1. INTRODUCTION

Geotechnical engineering always deals with risk and uncertainties in its field. These uncertainties are mostly tackled by probabilistic approach. One of the major applications is done to find out how reliable a given slope is in spite of the uncertainty of various parameters constituting that slope. The slope under Sonapur area (Assam) is taken under study in this paper.

The Sonapur is one the most problematic landslides in the North-East India. The Sonapur (latitude N 25°6'30" N and longitude E 92°21'51") slide is located at km 141.72 on NH-44 near Sonapur village; by the side of Lubha River. The landslide in this area became active during monsoon period in June 1988. Since 1988 the road gets blocked every year during monsoon period due to heavy flow of muck falling from a great height. There is used to be huge suffering on each monsoon due to closure of this important National Highway to approximately 16 Million populations of Tripura, Mizoram and Part of Assam and Manipur that cannot be quantified in financial terms. The photographs of sonapur landslide is given in Fig. 1.

Reliability method is a probability based approach of finding out whether a system can work satisfactorily under given condition.

In the 1970s the application of reliability methods in the analysis of slope stability (Wu and Kraft 1970; Catalan and Cornell 1976; Yang et al. 1977; Vanmarcke 1977) was initiated. This application has been continued till the present

age (Chowdhury, 1987; Lacasse, 1994; Christian, 1996; Hassan et al., 2000; El-Ramly et al., 2002).

Many reliability methods are developed over the years. They include Monte Carlo Simulation method (MCS), First and Second Order Reliability Methods (FORM, SORM), Mean-Value First-Order Second-Moment method (MFOSM). The results of their reliability analysis are expressed in terms of reliability index (β).

The uncertain parameters of a slope are found to be normally distributed so that all the analyses are based on normal distribution of data.

2. RELIABILITY METHODS

As mentioned above the different reliability processes are developed over the years. It is useful in dealing with the probabilities of failure or other measures of reliability. The engineer can incorporate uncertainty in the decision making process without establishing an absolute value of probability of failure. They are being discussed below.

2.1. Mean-Value First-Order Second-Moment method (MFOSM)

The mean-value first-order second-moment (MFOSM) method, also known as the first-order second-moment (FOSM) method, has long been applied to assess the reliability of soil embankments and slopes (Cornell, 1971).



Fig. 1: Photographs of Sonapur Landslide

It consists on a first order Taylor’s series approximation of the mean and variance of the performance function. More recently, a simple form of this method, referred to as the Mean-Value First-Order Second-Moment method (MFOSM), was proposed and used by Hassan *et al.*, (1999) to search for the critical slip surface with the minimal reliability index in a probabilistic slope stability analysis. An estimator of the Cornell reliability index, based on the MFOSM model, is given by:

$$\beta = \frac{G(\mu_{y_i})}{\sqrt{\sum_i^n (\frac{\partial G}{\partial Y_i})^2 \sigma_{y_i}^2 + 2 \cdot \sum_{i,j=1}^n \frac{\partial G}{\partial Y_i} \frac{\partial G}{\partial Y_j} \rho_{Y_i Y_j} \cdot \sigma_{Y_i} \cdot \sigma_{Y_j}}} \dots \quad 1.1$$

Where, β = reliability index, n = is the number of soil input parameters, μ_{y_i} the mean value of the uncertain parameter Y_i , σ_{y_i} and σ_{y_j} the standard deviation of Y_i and Y_j respectively, and $\rho_{y_i y_j}$ the coefficient of correlation between Y_i and Y_j .

It is straightforward to determine the probability of a slope failure using the MFOSM method. The key steps involve only the computation of the mean and the standard deviation of the factor of safety (FS) of slope. The mean Factor Safety, FS , is defined as the critical FS obtained using the mean values of all relevant parameters; while the standard deviation of FS, σ_{FS} , is obtained from the first terms of the Taylor series expansion (e.g., Baecher and Christian 2003) as follows:

$$\sigma_{FS}^2 = \sum_{i=1}^n \sum_{j=1}^n \left(\frac{\partial FS}{\partial X_i} \right) \left(\frac{\partial FS}{\partial X_j} \right) \rho_{X_i X_j} \sigma_{X_i} \sigma_{X_j} \dots \dots \dots 1.2$$

This study only considers FS to follow normal distribution, for which the reliability index, β , can be determined as,

$$\beta = \frac{FS-1}{\sigma_{FS}} \dots \dots \dots 1.3$$

And the probability of failure p_f is given by

$$P_f = 1 - \phi(\beta) \dots \dots \dots 1.4$$

Where, $\phi(\beta)$ is the standard cumulative distribution function.

2.2. Monte Carlo Simulation method

It was developed by John Von Neumann and Stanislaw Ulam when they published a paper, “The Monte Carlo Method” (1949). Based on the random concept of a slope stability Monte Carlo simulation is applied. First, a deterministic model is identified where multiple input variables are used to estimate an outcome of a single value. Then, a probability distribution for each independent variable is established for a simulation model. Next, a random trial process is initiated to establish a probability distribution for the deterministic situation being modelled.

Discrete values of the component random variables are generated with respect to their probability distribution, and the performance function is evaluated, then, for each generated set. The process is repeated many times to obtain an approximate, discrete probability density function of the

performance function. The MCS is the simplest and most robust method (Shreider, 1971); however, it is too time-consuming leading, hence, to prohibitive costs since a large number of realisations of each random variable. Y_i (parameter) is required to obtain accurate results, including the reliability index denoted β_{MCS} .

2.3. First and Second Order Reliability Methods (FORM, SORM)

The well-known Hasofer-Lind reliability index, proposed after by Hasofer *et al.* (1974), denoted by β_{HL} , is often preferred since it offers an invariant reliability measure. The determination of this index needs first, transforming the uncertain variable Y , into uncorrelated standard normal variable U . The design point is defined, in the standard normal space, as the point P^* that is located on the performance function ($G(U) = 0$). The Hasofer-Lind reliability index β_{HL} is defined by the distance from the origin to the design point in the standard normal space. In Fig. 4, this index is noted $E \beta_F$ and β_S associated respectively to FORM and SORM approximation methods.

3. SOFTWARES.

3.1 Geo5

GEO5 is a software suite for geotechnical analysis. It is consisted of individual applications, and each one solves a specific geotechnical problem. All programs are closely linked together and run in the same environment. GEO5 is designed to solve most common geotechnical tasks, as well as highly sophisticated problems, such as tunnel analysis, damage on building due to tunneling, stability of rock slopes etc. Staging the slope under different conditions the stability of the slope can be obtained for different given conditions. Varying the value of slope angle (θ), relative density (R.D.), water content (W.C.) and coefficient of friction (ϕ) factor of safety of the slope are obtained. Bishop’s analysis method is carried out for the optimized slip surface. Hence the FOS obtained is of the optimized value.

3.2. Plaxis 2D

PLAXIS 2D is a finite element package intended for the two dimensional analysis of deformation and stability in geotechnical engineering. It is equipped with features to deal with various aspects of geotechnical structures and construction processes using robust and theoretically sound computational procedures. With PLAXIS 2D the geometry of the model can be easily defined in the soil and structures modes, after which independent solid models can automatically be intersected and meshed. The staged construction mode allows for simulation of construction and excavation processes by activating and deactivating soil clusters and structural objects.

PLAXIS 2D is a user friendly geotechnical program offering flexible and interoperable geometry, realistic simulation of

construction stages, a robust and reliable calculation kernel, and comprehensive and detailed post-processing, making it a complete solution for geotechnical design and analysis.

3.3. Rt.

Rt is a computer program for reliability and optimization analysis with many probabilistic models. Rt is also fully parameterized, with individual objects for random variables, design variables, and model responses.

It allows the computation of reliability index using different processes of MCS, FORM/SORM, FOSM. It also gives the distribution curves a parameter.

4. LIMIT STATE FUNCTION

Typically, the soil strength is described by the Mohr-Coulomb failure criterion as a function of the cohesion, *c*, and the internal friction angle, ϕ . A common definition of the safety factor (*F*) is the ratio between the true shear strength of the material and the mobilized strength required for the limit equilibrium. Factor of safety for infinite slope without seepage is used

$$F = \frac{c}{\gamma H \cos^2 \beta \tan \beta} + \frac{\tan \phi}{\tan \beta} \dots\dots\dots 1.5$$

Where β = slope angle, γ = unit weight H= height of the slope.

5. LAB RESULTS

Direct shear test is done for finding the values of cohesion, *c*, and phi, ϕ . Tests were performed for different values of relative density, RD, and different water content, WC. The results are shown in table 1.

Table 1

RD	50%		60%		70%	
	c (Kpa)	phi(°)	c (Kpa)	phi(°)	c (Kpa)	phi(°)
0	20.5	27.16	21.1	30.37	22.1	33.38
3	20.4	26.2	20.9	29.1	21.8	32.2
5	20.2	25.45	20.7	28.77	21.5	31.92
8	19.8	25.1	20.5	28.2	21.2	31.5
10	19.5	24.75	20.4	28	20.9	31.13
12	19	24.1	20.2	27.99	20.7	30.77
15	18.4	23.7	20.2	27.83	20.5	30.37

Using the values above in table1, we can obtain the different values of mean and standard deviation for different RD, the corresponding coefficient of variation is also given in table 2.

Table 2

Parameters	Mean	Standard deviaton	COV	RD
c1 (kpa)	19.6	.775	.74	50%
ϕ 1 (°)	25.21	1.19		
c2 (kpa)	20.5	.345	.25	60%
ϕ 2 (°)	28.61	.90		
c3 (kpa)	21.24	.558	.496	70%
ϕ 3 (°)	31.61	1		

6. RESULTS OF THE ANALYSIS AND DISCUSSION

The results from the analysis as explained in the previous sections are obtained. Different reliability indices are obtained using combination of different possibilities of processes for reliability and limit state function.

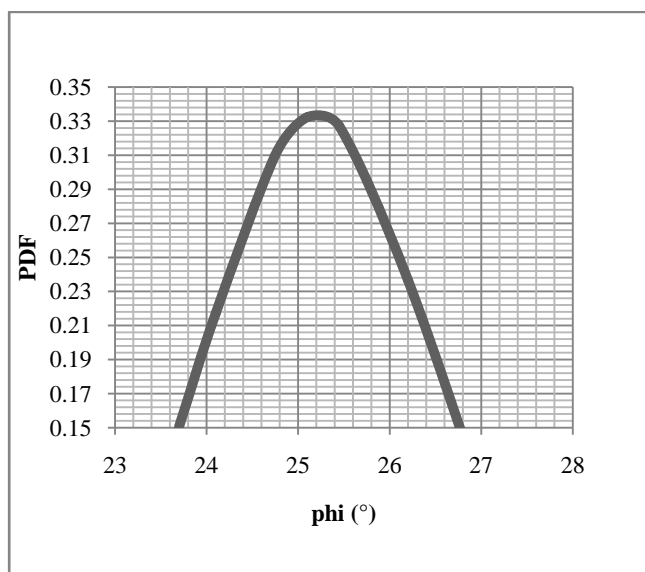


Fig. 2: Probability Distribution of Phi, ϕ .

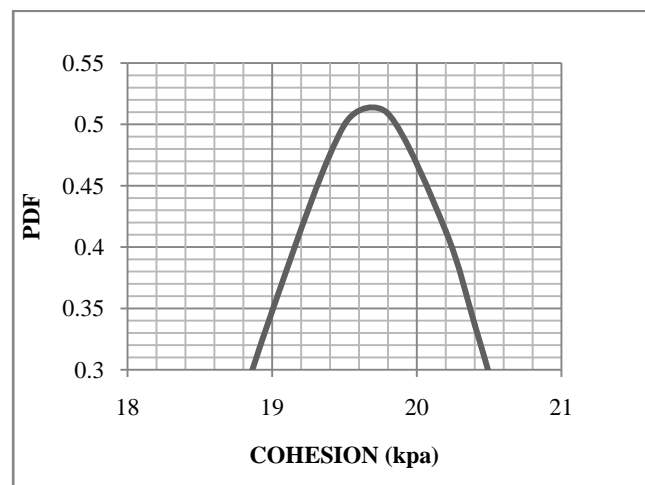


Fig. 3: Probability Distribution of Cohesion, c.

Fig. 1 and 2 show how c and ϕ follow a normal distribution hence all reliability analysis made here are based on the fact that the uncertain parameters are all normally distributed.

Applying MFOSM method, we can obtain the variation of reliability index β to probability of failure (p.o.f.), p_f as given in Fig. 3.

Fig. 3 clearly shows that with increase reliability index the value of probability of failure decreases, showing that the slope is becoming more safe with increasing reliability index

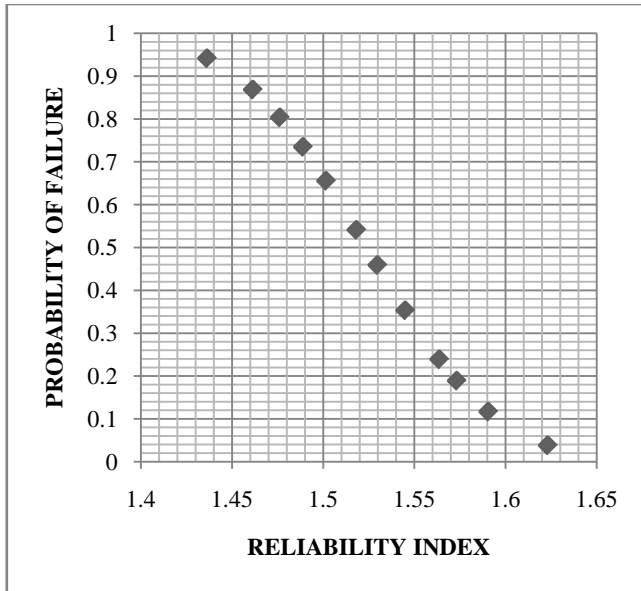


Fig. 4: Variation of reliability index β to p.o.f. p_f

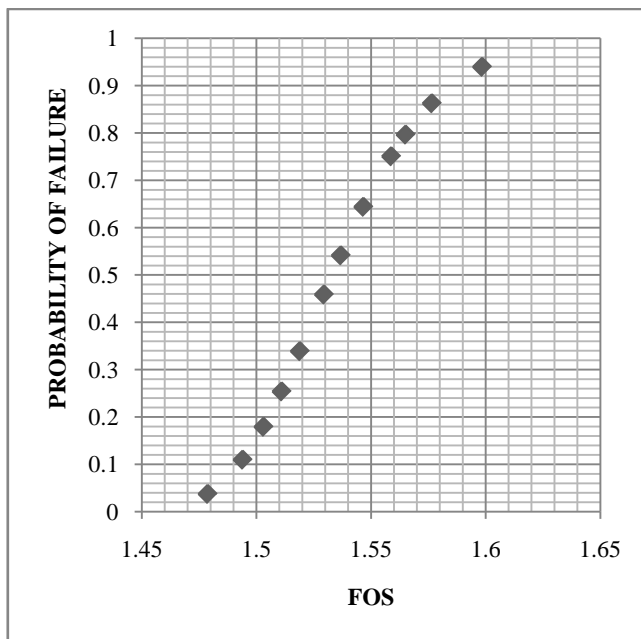


Fig. 5: Variation of FOS to the p.o.f. p_f

The variation of Factor of safety (FOS) to the probability of failure (p.o.f.), is given in Fig. 4. We can observe that as the FOS increase the probability of failure is also increasing.

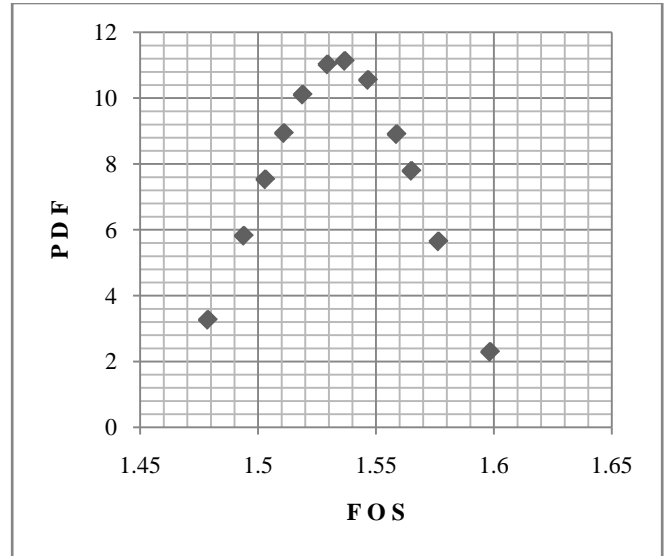


Fig. 6: Normal PDF of FOS (MFOSM)

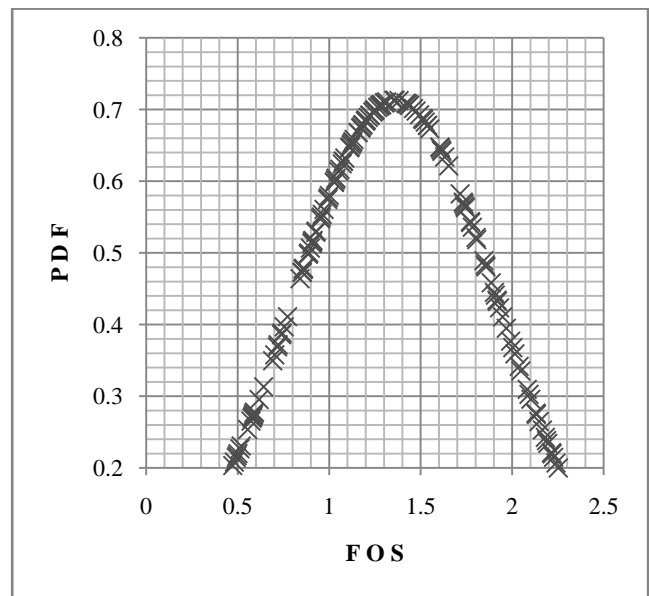


Fig. 7: Normal PDF of FOS (MCS)

As mentioned previously about the uncertain parameters being normally distributed, it is also observed that, ultimately, the FOS will also follow a normal distribution. This is shown in Fig. 5 and 6 for methods of MFOSM and MCS respectively.

The reliability index obtained from various combination of analysis are shown in table 2 and 3

Table 3: Manually obtained Reliability Index

Reliability method	Limit state function	Reliability index
MFOSM	Equation no.1.1	3.47
MCS	GEO5	2.404
	PLAXIS 2D	4.302
FOSM	GEO5	2.224
	PLAXIS 2D	2.76

Table 4: Reliability index from Rt.

Reliability method	Limit state function	Reliability index
FOSM	GEO5	2.207
	PLAXIS 2D	2.768
MCS	GEO5	2.41
	PLAXIS 2D	4.26
FORM/SORM	GEO5	2.207
	PLAXIS 2D	2.96

Table 1 gives the values of reliability index obtained when calculated manually, and table 2 gives the values when inputs are given in Rt.

When comparing the index for MCS obtained from both processes (manually and Rt), it can be observed that there is a good similarity between these values

For FOSM the index are almost of the same values for both differently obtained safety factor of slope.

It can be noted herein the importance of a reliability slope stability analysis, not as a substitute, but as a complementary analysis to the deterministic one. The reliability approach can characterize and incorporate the variability of soil properties into the analytical process. As a result, most of the test data is implicitly used in the analysis. The resultant reliability index contains more information than the deterministic safety factor.

Table 5: Reliability Index (β) for different water content (WC).

WC	0	3	5	8	10	12	10
β	2.21	2.189	2.199	2.188	2.189	2.188	2.23

Table 5 shows that increasing in water content the reliability index decrease. Hence it can be said that the reliability of the slope decreases as the water content increase hence increasing the probability that the slope will fail.

7. CONCLUSIONS

In this paper, reliability of a homogeneous slope is investigated by means of different probabilistic methods, including MCS, FORM/SORM and MFOSM. As part of this effort, three mechanical models were considered: one from the general Factor of Safety formula for infinite slope, the second using the values obtained by taking the optimized slip surface in bishop's method, and thirdly using the finite element method using PLAXIS 2D.

Soil cohesion and friction angle are taken as the uncertain parameters. In addition to the reliability index and probability of failure, Rt gives the probability distribution curve and the value of probability of failure. It also shows how the number of samples varies with the coefficient of variation in MCS.

Results of these reliability slope stability analyses show that the factor of safety is not considered as the only term to express safety of a slope in a reliability analysis.

As mentioned before Sonapur landslide occurs during rainy season. From the study we can see how reliability index decreases as the water content increase. Hence we can conclude that the reliability of Sonapur slope decreases during monsoon period, hence making the slope more susceptible to landslides at this season.

The change in soil condition leads to the change in the reliability of the soil. When the water content of the soil is more soil tends to fail rapidly when compare to the dry soil.

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